Name:

No notes or other aids are allowed. Read all directions carefully and write your answers in the space provided. To receive full credit, you must show all of your work. The final answer without steps/work is only worth 0.25 of the points of the question

1. (7 points) Evaluate the following integral using a trigonometric substitution.

$$\int \frac{1}{x^2 \sqrt{x^2 + 16}} \, dx$$

2. (3 points) Write down the general form of the partial fraction decomposition of the following integral. You are **not required** to solve for the constants in the numerator. **Don't integrate.**

$$\int \frac{x+3}{x^2(x^2+1)^2} \, dx$$

3. (6 points) Evaluate the following integral using partial fraction

$$\int \frac{1}{(x-1)(x^2+1)} \, dx$$

4. (a) (4 points) Approximate the value of

$$\int_{1}^{6} \frac{dx}{x^3}$$

with n = 5 subintervals to 6 decimal places using Trapezoidal Rule.

(b) (3 points) How large should n be in order to guarantee that the Trapezoidal Rule estimate for $\int_{1}^{6} \frac{dx}{x^3}$ is accurate to within $0.00001 = 10^{-5}$?

5. (5 points) Determine if the following integral converges of diverges. Explain why.

$$\int_{e}^{\infty} \frac{1}{x(\ln(x))^3} \, dx$$

6. (5 points) Compute the arc length of the curve $f(x) = \ln(\sin x)$ over the interval $[0, \pi/4]$.

7. Extra credits (**optional**) (4 points) Use the **comparison theorem** to determine whether the improper integral

$$\int_1^\infty \frac{x^2}{\sqrt{x^8 + 1}} \, dx$$

is convergent or divergent (Show your work!).